

# Wave Star Energy

Wave Star bølgekraftmaskine 1:40 skala model

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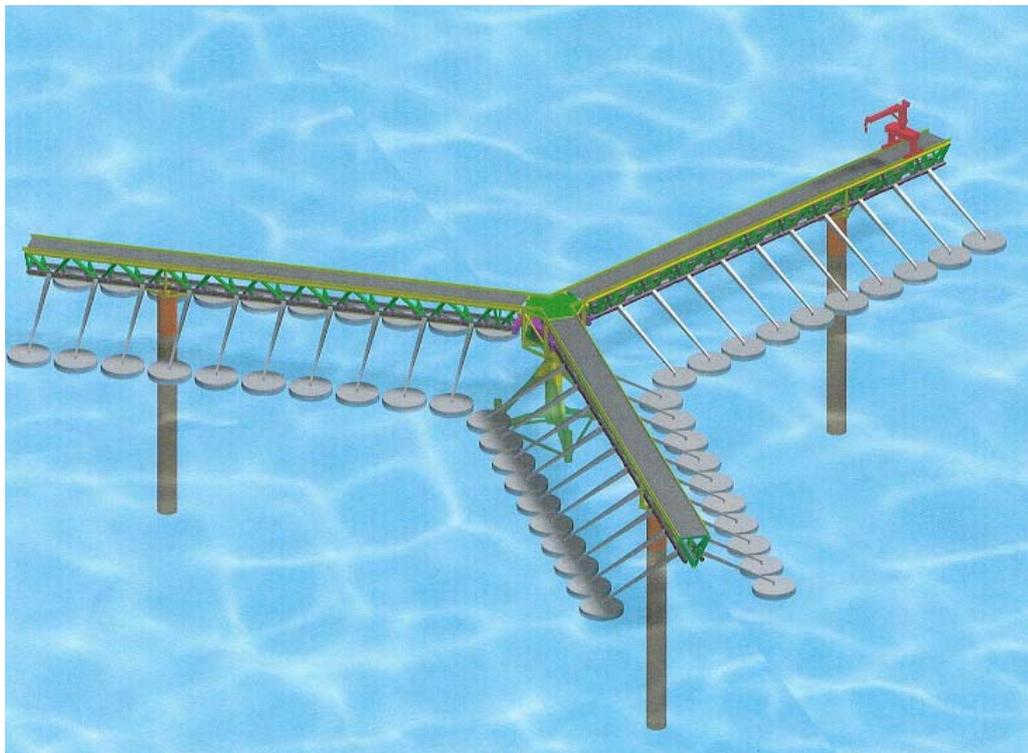
## Table of contents

|                |  |           |
|----------------|--|-----------|
| <b>1.</b>      | <b>Introduction</b>  | <b>1</b>  |
| 1.1            | The principle of operation                                 | 1         |
| <b>2.</b>      | <b>Objectives</b>  | <b>3</b>  |
| <b>3.</b>      | <b>Results</b>   | <b>4</b>  |
| 3.1            | Power calculations in regular waves:                       | 4         |
| 3.2            | Power production in irregular waves                        | 6         |
| 3.3            | Sensitivity to wave period in irregular sea                | 7         |
| 3.4            | Array interactions   | 10        |
| <b>4.</b>      | <b>Conclusions</b>   | <b>13</b> |
| <b>5.</b>      | <b>Methodology</b>   | <b>14</b> |
| 5.1            | Linear theory for heaving motion of the float              | 14        |
| <b>Annex I</b> | <b>WaveStar multi-float system performance calculation</b> |           |

# 1. Introduction

The development of the *Wave Star* aims at integrating known technology and applying offshore technology in an innovative way.

The present report is prepared for Wave Star Energy with the financial support from the PSO funding programme. Rambolls task in this project is to provide an independent assessment on the energy production based on well-documented numerical modelling techniques, integrating the hydrodynamic coefficients related to the float geometries calculated and reported by Aalborg University [1].



*Figure 1 Artist impression of the Wave Star wave energy converter.*

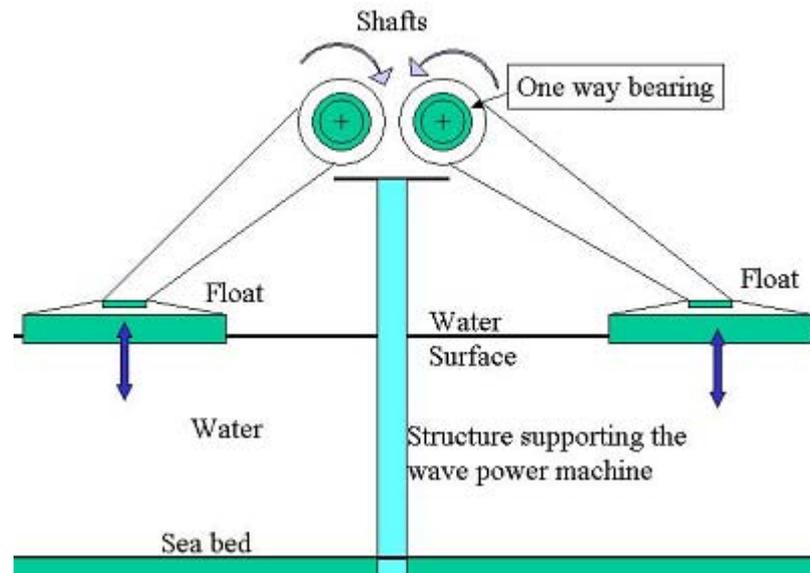
The numerical models describe the operation of the system in normal wave conditions and it is expected that in stormy conditions, the generator is cut off, and the floats move, without applying significant forces on to the structure.

The mechanical structure is placed sufficiently high above the water surface, so even the highest waves cannot reach the structure.

## 1.1 The principle of operation

The machine is based on rows of floats which are all fixed by leavers to a horizontal shafts, with one-way bearings. The slowly revolving shafts are connected through a gearbox to a generator, similar to the method used in a wind turbine. The length of the row of floats is at least a wavelength. When a wave passes, the first

float is lifted upwards, because of the buoyancy, and the lever locks onto the shaft, when the speed exceeds the revolving speed of the shaft. This generates a torque to the shaft. When the wave descends, the grip of the shaft is loosened and the float moves freely down to a lower position until the next wave appears. The next float in line, adds a similar torque to the shaft. The shaft in this way integrates the torque from all upward moving floats at a fixed rotational speed – a speed that is determined by the gearing and the generator speed.



*Figure 2. Principle of operation*

On the opposite side, there is a similar row of floats and an additional shaft, which is driven in the opposite direction.

## 2. Objectives

The objectives of the present report are to calculate the energy production from one float and from a row of five floats for waves with different directions of incidence. It is the objective to use well-documented numerical modelling techniques, and to make the results comparable. The hydrodynamic coefficients related to the float geometries has been calculated and reported by Aalborg University [1].

The following tasks are included

1. Power calculation in regular waves

Calculate the average power produced by a single float of diameter 10 meter in a regular 2 meter high wave with wave periods ranging from 4 sec. to 12 sec. This task should be seen as a reference i.e. for comparison with experimental results in regular waves. In theory the power produced in regular waves of a certain wave period is proportional the wave height squared and the results presented for 2 meter can be extrapolated to any wave height.

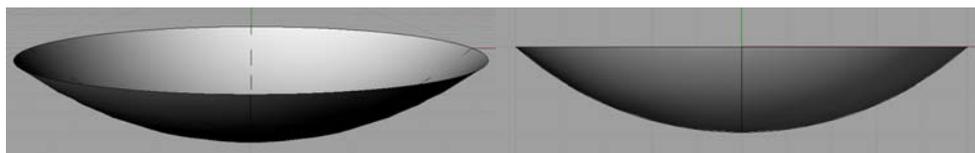
2. Power calculations in irregular sea conditions

Calculate the average power produced by a single float of diameter 10 in a irregular sea condition with a significant wave height of 2.5 m and average wave period of 5.5 sec. Further calculate the sensitivity to wave periods and the average power production in sea conditions corresponding to the Danish part of the North Sea.

3. Array interaction in irregular sea conditions

Calculate the power produced by an array of 5 floats of diameter 10 with incident waves of direction  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , and  $90^\circ$  and with gabs between the floats of 0, 2m, 5m and 10m respectively. The power calculations for the array is carried out in the same sea condition as task 2 with a significant wave height of 2.5 m and average wave period of 5.5 sec.

For all calculations the same float geometry is used. The float geometry has been selected by Aalborg University based on the report on the wave exiting force and hydrodynamic coefficients. The report contains data for a few different float geometries. The float geometry selected for the present report is shown on figure 3 below. The float diameter is 10 meter in its water plane, spherically shaped with a radius of 7,5 m. and a volume of  $V = 78,5 \text{ m}^3$ , a draught of 1,91 m. [2].



*Figur 3. Float geometry from AUC [2].*

### 3. Results

The results presented in this study are based on hydrodynamic calculations using linear first order theory as described in chapter 5.

#### 3.1 Power calculations in regular waves:

The results in terms of power generated by a single float in regular waves are shown in the table 1 below. The wave height is 2 meter and the wave period is varied from 4 sec. to 12 sec. In each wave situation the applied damping has been optimised in order to extract most energy. The amount of damping is expressed by the damping factor  $r_d$ .

|              |      |      |      |      |      |
|--------------|------|------|------|------|------|
| T [sec]      | 4    | 6    | 8    | 10   | 12   |
| H [m]        | 2    | 2    | 2    | 2    | 2    |
| z [m]        | 0,92 | 1,22 | 1,32 | 1,38 | 1,38 |
| $r_d$        | 0,85 | 1,83 | 3,01 | 4,12 | 5,27 |
| P [kW]       | 56   | 94   | 103  | 97   | 88   |
| $P_w$ [kW/m] | 16   | 24   | 31   | 39   | 47   |
| CWR [%]      | 34   | 40   | 33   | 25   | 19   |

**Table 1. Power calculations in regular wave height of 2 meter.**

|       |         |  |
|-------|---------|--|
| T     | [sec]   | Wave period                                  |
| H     | [meter] | Wave height                                  |
| z     | [meter] | Float excursion (lowest to highest position) |
| $r_d$ |         | Damping factor                               |
| P     | [kW]    | Generated power                              |
| $P_w$ | [kW/m]  | Incident power per unit length               |
| D     | [m]     | Float diameter                               |
| CWR   | [%]     | Capture width ratio                          |

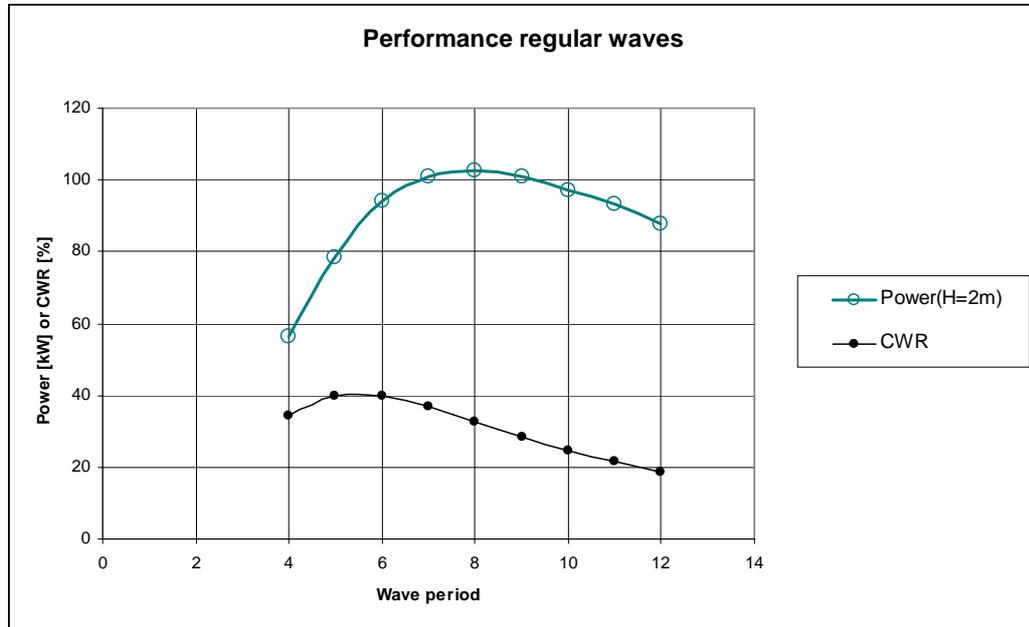
The incident power in regular waves can be calculated as:

$$P_w = \frac{\rho g^2}{32\pi} H^2 T$$

The Capture With Ratio is calculated as:

$$CWR = \frac{P}{D \cdot P_w}$$

The calculations show that the float is able to absorb about 100 kW of power from the 2 metre high waves in quite a wide range of wave periods from 6 seconds up to 10 seconds if optimal damping is applied.



*Figure 4. Power production in regular waves of height 2 meter and capture with ratio.*

The results in terms of capture with ratio show that the single float will absorb power from a wave crest that is about 40 % of the float diameter. As the wave periods become longer the power in the wave increase proportionally to the wave period, but as the power output remains the same the CWR decrease to about 20%.

### 3.2 Power production in irregular waves

In the real sea the waves are not regular but irregular. Short and longer waves are mixed and wave heights seem to vary randomly and appear in groups.

The energy production in irregular wave conditions however can be calculated using the same mathematical model as in regular waves, but applying a spectrum to describe the distribution of energy on different frequencies in the sea state. The sea state is defined from two parameters the significant wave height  $H_s$  and the average wave period  $T_z$ .

$H_s$  [m]      Significant wave height

$T_z$  [sec]      Average wave period

In the North Sea a central estimate of the average wave period  $T_z$  for each significant wave height  $H_s$  shows a relation

$$T_z = H_s \cdot \text{sec/m} + 3 \text{ sec}$$

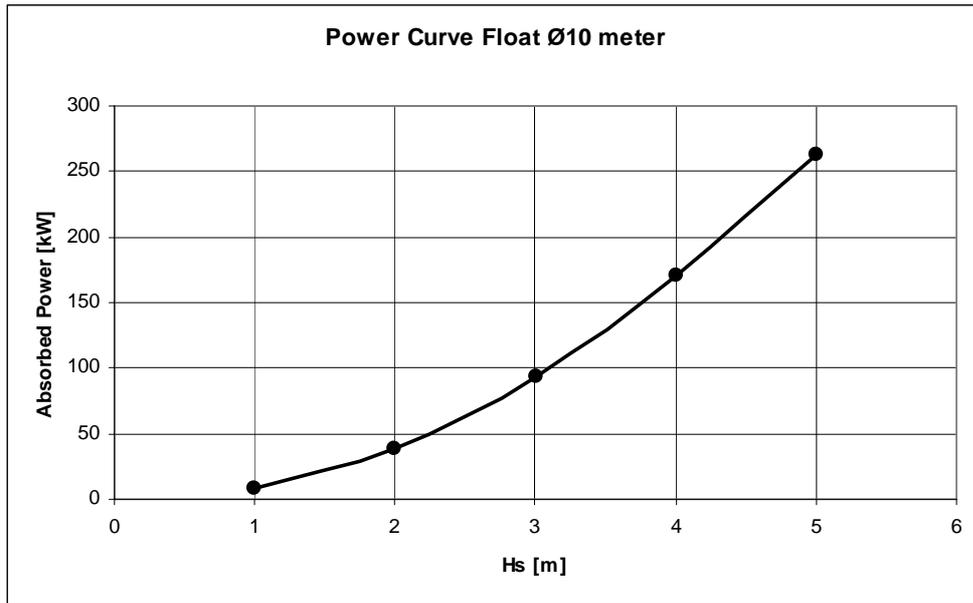
|                |      |      |      |      |      |      |       |
|----------------|------|------|------|------|------|------|-------|
| $H_s$ [m]      | <0.5 | 1,0  | 2,0  | 3,0  | 4,0  | >4,5 |       |
| $T_z$ [sec]    |      | 4    | 5,0  | 6,0  | 7,0  | 8,0  |       |
| $r_d$          |      | 1,6  | 2,1  | 2,6  | 3,2  | 3,8  |       |
| $P_{abs}$ [kW] | -    | 8    | 39   | 94   | 171  | 263  |       |
| Hours / year   | 966  | 4103 | 1982 | 944  | 445  | 330  | Total |
| MWh / year     |      | 32,6 | 76,7 | 89,1 | 76,1 | 86,8 | 361   |

**Table 2. Power calculations for dominating sea states in the North Sea, indicating the number of hours each sea state is prevailing.**

For each of these five sea states the theoretical absorbed power is calculated and multiplied with the number of hours the sea state prevails. Summing up the contribution from each sea state the 10-meter float in theory can produce 361 MWh/Year.

If the rated power of one float is 263 kW then the full load hours (FLH) will amount to  $FLH = 361.000 / 263 = 1387$  hours

If the rated power is reduced say at 120 kW i.e. by applying non-optimal damping in the larger sea states than the energy production of-course will be reduced but the full load hours increased. This can be a useful strategy when calculating the overall cost including the cost of generator and transmission equipment.



*Figure 5. Theoretical power curve for a single 10-meter diameter float*

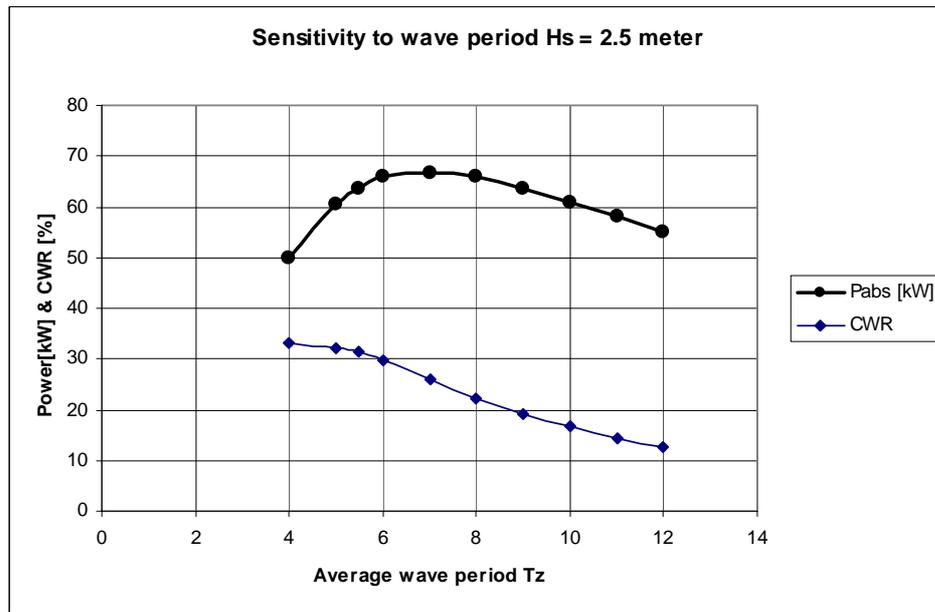
Figure 5 shows the power curve for a single float of the wave power converter. The generated power is shown as a function of the significant wave height. This curve is similar to the power curves for wind turbines providing information of power production as a function of the wind speed.

### 3.3 Sensitivity to wave period in irregular sea

The sensitivity to wave periods in irregular sea is calculated for a constant significant wave height of 2,5 meter and wave periods ranging from 4sec. to 12 sec. as shown in Table 3 and graphically on the Figure 6. The calculations shows that the system is not very sensitive to the wave period, however the damping is adjusted to optimum power production at each wave period. The long the wave periods require more damping compared to short wave periods in order to achieve optimum power production. Higher damping is similar to larger torque on the shaft or larger reacting force in the hydraulic rods opposing the motion of the floats.

|                 |      |      |      |      |      |      |
|-----------------|------|------|------|------|------|------|
| Tz [sec]        | 4    | 5,5  | 6    | 8    | 10   | 12   |
| Hs [m]          | 2,5  | 2,5  | 2,5  | 2,5  | 2,5  | 2,5  |
| $r_d$           | 1.33 | 2,44 | 2,57 | 3,75 | 4,9  | 6,3  |
| $P_{abs}$ [kW]  | 50   | 63,6 | 66   | 66   | 61   | 55   |
| $P_{wr}$ [kW/m] | 14,7 | 20,2 | 22   | 29,5 | 36,8 | 44   |
| CWR [%]         | 33   | 31   | 30   | 22   | 16,6 | 12,5 |

**Table 3. Power calculations in regular wave height of 2,5 meter**



**Fig 6. Power production in a sea state with significant wave height  $H_s = 2,5$  meter at different average wave periods  $T_z$  and variation of Capture with ratio in irregular sea.**

The power produced depending on the wave period in irregular waves shows the same tendency as in regular waves. The float produces optimum amount of power in wave periods between 6 and 8 sec and the system is not very sensitive to variations in wave periods

At small wave periods 2-4 sec. the vertical force is reduced, as the short waves does not “cover the float”, at longer wave periods above 9 sec. the time interval between each power stroke becomes longer and the wave force goes toward a constant value (the buoyancy force) and the asymptotic behaviour of the power production becomes inversely proportional to the wave period.

The power in each irregular sea state is about the same as the corresponding power in the regular wave of same period. However the captured power is reduced from 100 kW to about 66 kW, this is a result of the irregular nature of the waves.

From the calculations above it is possible to present a power matrices that provide more information on the system performance in a wide range of combinations of  $H_s$  and  $T_z$ . In theory the power produced will vary proportional to the  $H_s$  squared for a given value of  $T_z$ .

|  |   | T <sub>z</sub> [sec] |    |     |     |     |     |
|--|---|----------------------|----|-----|-----|-----|-----|
|  |   | 4                    | 5  | 6   | 7   | 8   | 9   |
| Significant wave height H <sub>s</sub> [m] | 1 | 8                    | 10 | 10  | 11  | 11  | 10  |
|  | 2 | 32                   | 39 | 42  | 43  | 42  | 41  |
|  | 3 |                      | 88 | 94  | 96  | 95  | 92  |
|  | 4 |                      |    | 167 | 171 | 168 | 163 |
|  | 5 |                      |    |     | 267 | 263 | 254 |

**Table 4. Power matrices provide information on the power production in kW in different combinations of H<sub>s</sub> and T<sub>z</sub>.**

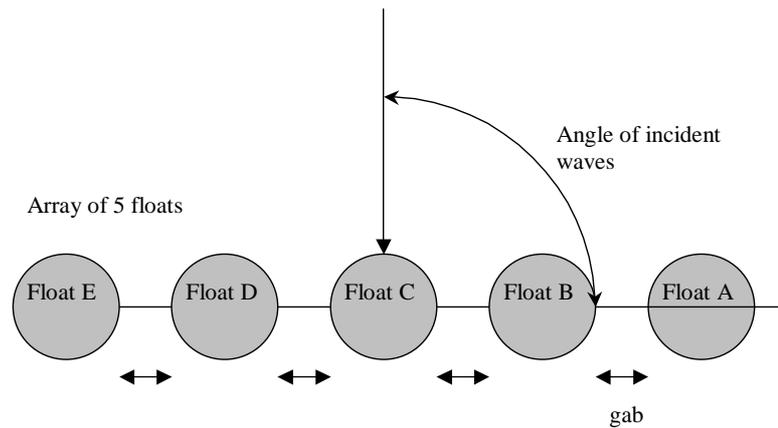
The presentation on power production for wave energy systems has not yet been standardised. Due to the fact that some sea locations in other parts of the world have a much wider span of wave periods compared to the Danish Part of the North Sea and Wave Power systems are much more sensitive to wave periods compared to the present system, power matrices have been suggested as a possible way to characterise the performance of the converter.

The performance matrices as shown in table 4 is the theoretical power production and does not include conversion efficiencies associated with losses in the PTO system. Once the PTO system has been developed the conversion efficiency in each cell can be determined.

For a single float the power matrices is independent of the wave direction. For the fully developed system matrices must be established for each direction of incidence. The next section will look at the variation of performance for an array of five floats with incoming waves from different angles.

### 3.4 Array interactions

The array interactions have been calculated in a similar way as for one float in irregular waves (above). That is the theory is linear and the applied damping has been optimised. The purpose of these calculations has been to evaluate how much the average absorbed power by each float will change if several floats are placed side by side (90°) - or if the floats are placed one after the other (0°) so that the first float experience the full wave and the floats behind are a bit in the shelter of the first.



**Figure 7. Definition of incident wave direction – (If the angle of incidence is 90 degree then the floats in long crested waves will move identically up and down.)**

The array interaction is described as the quantity called the q-factor or the magnification factor, which can be above or below 1 depending on constructive or destructive array interaction. The “q-factor” express the average power absorbed by the array (of five floats), compared to the power absorbed by (five) isolated floats:

$$q = \frac{P_{abs}(array)}{P_{abs}(onefloat) * n}$$

The q factor sums up the interaction as the result of the diffraction force (that is the wave exiting force) that will be experienced by each float and the reflections from the other floats if the array is held fixed in the incoming waves. The other part is the wave exiting forces the other floats generate by their movements – if one float starts to move in still water it will generate waves that will case the other floats to move.

The results of the calculations are shown on the table below and the absorbed power of each member in the array is indicated. The average overall power for the array is shown as well. These calculations have been performed in a wave situation of  $H_s = 2.5$  meter and  $T_z = 5.5$  sec and the absorbed power from a single float is 63,6 kW

| Gab (m) | Wave Diction | Float A (kW) | Float B (kW) | Float C (kW) | Float D (kW) | Float E (kW) | Average (kW) | q factor |
|---------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|----------|
| 0       | 0            | 79,6         | 51,7         | 44,6         | 37,7         | 29,4         | 48,60        | 0,76     |
| 0       | 30           | 75,6         | 50,7         | 45,6         | 40,5         | 32,9         | 49,05        | 0,77     |
| 0       | 45           | 70,7         | 50,4         | 48,4         | 47,7         | 41,2         | 51,66        | 0,81     |
| 0       | 60           | 65,7         | 51,5         | 53,1         | 57,2         | 52,0         | 55,88        | 0,88     |
| 0       | 90           | 63,1         | 59,5         | 59,3         | 59,5         | 63,1         | 60,91        | 0,96     |
|         |              |              |              |              |              |              |              |          |
| 2       | 0            | 74,0         | 50,5         | 45,5         | 41,2         | 31,7         | 48,58        | 0,76     |
| 2       | 30           | 70,4         | 49,0         | 47,2         | 45,6         | 39,0         | 50,23        | 0,79     |
| 2       | 45           | 66,0         | 48,7         | 51,4         | 54,2         | 49,7         | 54,00        | 0,85     |
| 2       | 60           | 61,8         | 49,8         | 56,8         | 65,1         | 61,2         | 58,95        | 0,93     |
| 2       | 90           | 64,8         | 62,2         | 63,5         | 62,2         | 64,8         | 63,49        | 1,00     |
|         |              |              |              |              |              |              |              |          |
| 5       | 0            | 71,1         | 52,8         | 47,6         | 44,9         | 35,3         | 50,36        | 0,79     |
| 5       | 30           | 68,6         | 52,1         | 50,7         | 53,0         | 45,1         | 53,90        | 0,85     |
| 5       | 45           | 65,5         | 52,7         | 56,8         | 63,0         | 59,9         | 59,60        | 0,94     |
| 5       | 60           | 62,3         | 54,9         | 64,6         | 73,8         | 70,2         | 65,15        | 1,02     |
| 5       | 90           | 67,2         | 69,9         | 73,5         | 69,9         | 67,2         | 69,51        | 1,09     |
|         |              |              |              |              |              |              |              |          |
| 10      | 0            | 67,4         | 55,3         | 49,6         | 46,7         | 39,9         | 51,79        | 0,81     |
| 10      | 30           | 66,4         | 56,5         | 54,7         | 55,7         | 52,8         | 57,21        | 0,90     |
| 10      | 45           | 64,8         | 59,0         | 62,5         | 69,6         | 70,2         | 65,20        | 1,03     |
| 10      | 60           | 63,1         | 62,7         | 70,8         | 79,7         | 77,7         | 70,81        | 1,11     |
| 10      | 90           | 67,9         | 75,1         | 77,4         | 75,1         | 67,9         | 72,71        | 1,14     |

**Table 5. Results of array calculations for 5 floats in sea state  $H_s = 2.5$  m and  $T_z = 5.5$  sec**

From the table 5 one can see that the array of floats will absorb less power compared to five single floats if the floats are placed behind each other so that float A gets the wave first then float B etc. (Wave direction 0) The average reduction is about between 0.76 and 0.81 least reduction if the gab is 10 meter. If the waves hit the array of floats from the side (Wave direction 90) so all floats move up and down at the same time - then there is a constructive interaction if the gab is greater than 2 meter – the average power is increased between 1 – 1.14 compared to five single floats.

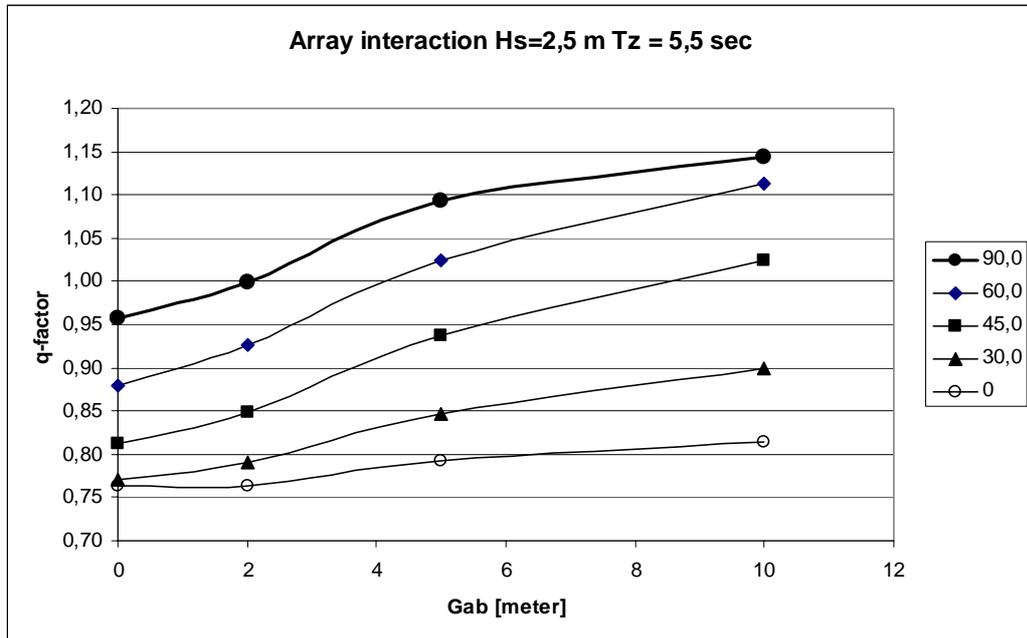


Figure 8. Plot of the results of the array calculations showing the q-factor as a function of the gab between the floats for different angles of incidence.

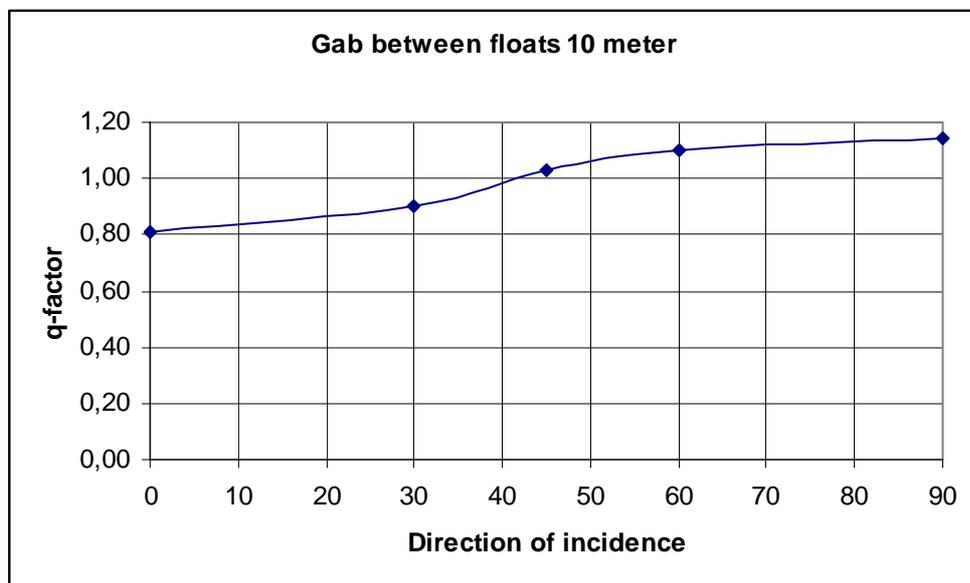


Figure 9. Array interaction between floats

## 4. Conclusions

The results presented on power produced by a single float end floats in array using linear theory are expected to be an upper limit to what is achievable. The results do not take into account energy losses in the power conversion system or in the transmission system.

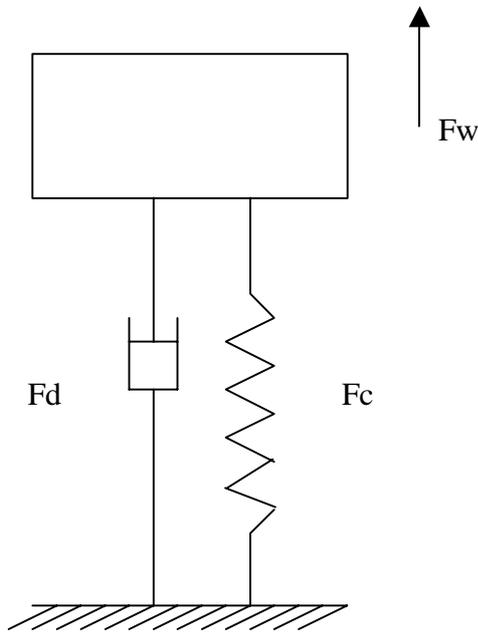
The conclusions to the study are that the array interaction and spacing is important to consider when designing the system.

For the present study only 5 floats have been considered and the design of the wave star includes two lines of 20 floats. The ongoing testing at Aalborg University can give some indication to the overall absorbed power from such a construction, however reflections from the basin walls can also influence the results.

In order to avoid the influence of reflections from the basin walls it is obvious that open sea testing can provide such results. In open sea however the measurements of the incoming waves has to be given some consideration as where to measure these as reflected waves from the device will be measured as well depending on the distance between the wave measurements and the device.

## 5. Methodology

### 5.1 Linear theory for heaving motion of the float



*Fig 10. Principle of oscillating float system*

The linear model of the system is introduced as its possible to solve analytically. In its basic formulation the equation of motion for the forced oscillating system is:

$$[M + a(\omega)]\ddot{z} + [b(\omega) + d(\omega)]\dot{z} + Sz = F_w(\omega t)$$

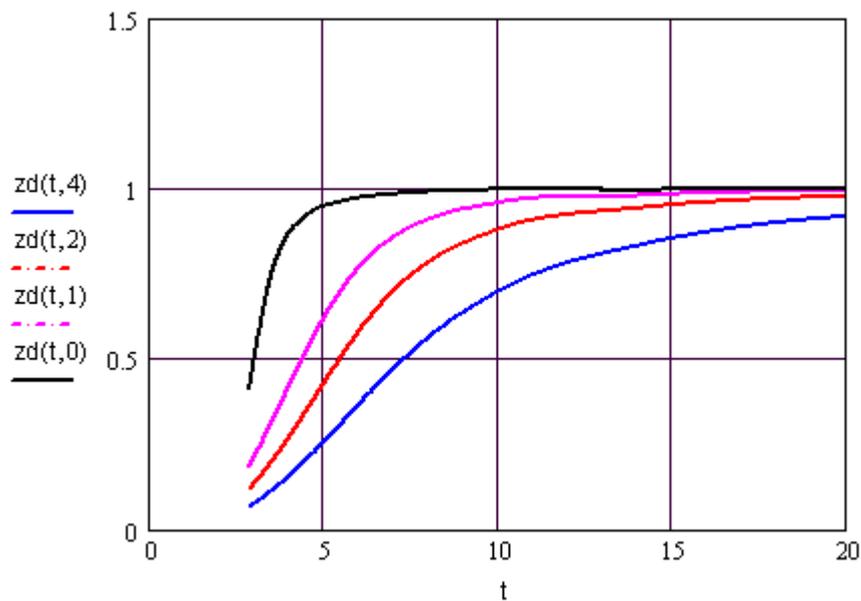
|             |                        |  |
|-------------|------------------------|--|
| $M$         | [kg]                   | Float mass   |
| $a(\omega)$ | [kg]                   | Added mass (hydrodynamic)                              |
| $b(\omega)$ | [kg/s]                 | Damping (hydrodynamic)                                 |
| $d(\omega)$ | [kg/s]                 | Mechanical damping (PTO)                               |
| $S$         | [kg/s <sup>2</sup> ]   | Restoring stiffness (associated with water plain area) |
| $F_w$       | [kg*m/s <sup>2</sup> ] | Wave exciting force (hydrodynamic)                     |
| $\omega$    | [rad/sec]              | Wave frequency   |
| $t$         | [sec]                  | Time   |
| $z$         | [m]                    | Float position   |
| $\dot{z}$   | [m/s]                  | Float velocity   |
| $\ddot{z}$  | [m/s <sup>2</sup> ]    | Float acceleration                                     |

The damping is the mechanical force that extract the power from the wave induced motion on the float. Using linear theory the damping force is proportional to the float velocity. The damping coefficient that provides the optimal power production depends on the wave period.

If there is no external damping on the system you obtain the free amplitude response of the float to waves of different wave periods.

The Response Amplitude Operator is defined as:

$$RAO = \frac{|z_f|}{A}$$



*Figure 11. Float response as function of wave period for different values of external damping.*

The response from the light float is always equal or below 1.0 compared to the response of a float with twice the oscillating mass the heavier float shows a resonance peak at about 5 second waves.

The more damping the less movement of the float however the product of movement per wave cycle times the damping force gives the produced power P.

The Capture Width Ratio is an expression used to indicate how much energy the float can absorb. If the capture is 1 it means that the float absorbs the energy contained in a wave front of same width as the float diameter.

$$CWR = \frac{P(\omega)}{D \cdot P_w(\omega)}$$

For the light float the capture width ratio will depend on the amount of external damping is imposed on the float.

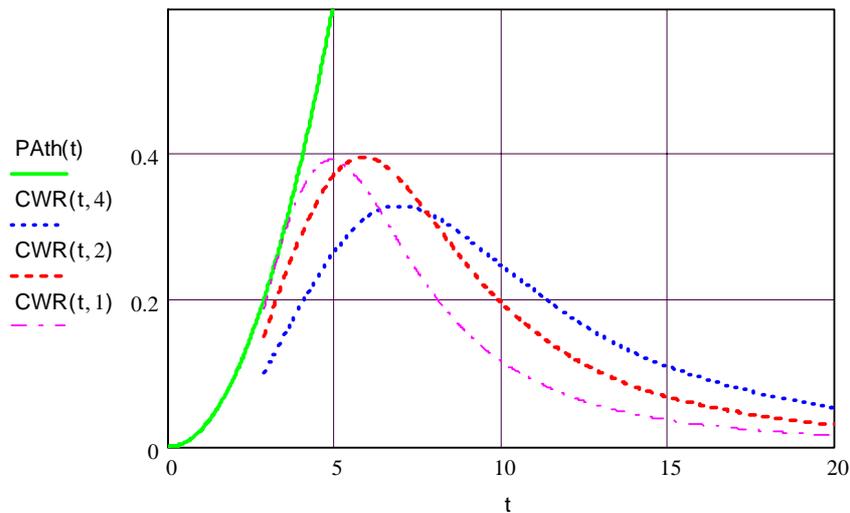
Point absorber theory indicates that a float optimally controlled should be able to absorb power equal to  $\lambda/2\pi$  independent of the float diameter.

$\lambda$  [m]          Wave length

This corresponds to a theoretical capture width ratio of point absorbers become:

$$PAth(T) = \frac{P_w(t) \cdot \lambda / 2\pi}{P_w(t) \cdot D} = \frac{gT^2}{(2\pi)^2 \cdot D}$$

The present point absorber is a non resonant point absorber and the amplitudes in general smaller than the wave amplitudes. Therefore the system reaches the theoretical limit at very short waves and never captures more energy than what is contained in a wave front about 40% of the float diameter. However it is worth to notice that this “low” energy capture is maintained over a wide range of wave periods.



**Figure 12.** Capture width ratios CWR as function of wave period for different values of the external damping. The green curve shows the theoretical capture width ratio for Point Absorbers.